7.20. **IDENTIFY:** Use energy methods. There are changes in both elastic and gravitational potential energy. **SET UP:** \( K_1 + U_1 + W_{\text{other}} = K_2 + U_2 \). Points 1 and 2 in the motion are sketched in Figure 1.

![Figure 1](image)

The spring force and gravity are the only forces doing work on the cheese, so \( W_{\text{other}} = 0 \) and \( U = U_{\text{grav}} + U_{\text{el}} \).

**EXECUTE:** Cheese released from rest implies \( K_1 = 0 \).

At the maximum height \( v_2 = 0 \) so \( K_2 = 0 \).

\[
U_1 = U_{1,\text{el}} + U_{1,\text{grav}}
\]

\( y_1 = 0 \) implies \( U_{1,\text{grav}} = 0 \)

\[
U_{1,\text{el}} = \frac{1}{2} k x_1^2 = \frac{1}{2} (1800 \text{ N/m})(0.15 \text{ m})^2 = 20.25 \text{ J}
\]

(Here \( x_1 \) refers to the amount the spring is stretched or compressed when the cheese is at position 1; it is not the \( x \)-coordinate of the cheese in the coordinate system shown in the sketch.)

\( U_2 = U_{2,\text{el}} + U_{2,\text{grav}} \)

\( U_{2,\text{grav}} = mg y_2 \), where \( y_2 \) is the height we are solving for. \( U_{2,\text{el}} = 0 \) since now the spring is no longer compressed.

Putting all this into \( K_1 + U_1 + W_{\text{other}} = K_2 + U_2 \) gives \( K_{1,\text{el}} = U_{2,\text{grav}} \)

\[
y_2 = \frac{20.25 \text{ J}}{mg} = \frac{20.25 \text{ J}}{(1.20 \text{ kg})(9.80 \text{ m/s}^2)} = 1.72 \text{ m}
\]

**EVALUATE:** The description in terms of energy is very simple; the elastic potential energy originally stored in the spring is converted into gravitational potential energy of the system.

7.30. **IDENTIFY** and **SET UP:** The friction force is constant during each displacement and Eq.(6.2) can be used to calculate work, but the direction of the friction force can be different for different displacements.

\[
f = \mu mg = (0.25)(1.5 \text{ kg})(9.80 \text{ m/s}^2) = 3.675 \text{ N}; \text{ direction of } \vec{f} \text{ is opposite to the motion.}
\]

**EXECUTE:** (a) The path of the book is sketched in Figure 2a.

![Figure 2a](image)

For the motion from you to Beth the friction force is directed opposite to the displacement \( \vec{s} \) and \( W = -fs = -(3.675 \text{ N})(8.0 \text{ m}) = -29.4 \text{ J} \).
For the motion from Beth to Carlos the friction force is again directed opposite to the displacement and \( W_2 = -29.4 \text{ J} \).

\[
W_{\text{tot}} = W_1 + W_2 = -29.4 \text{ J} - 29.4 \text{ J} = -59 \text{ J}
\]

(b) The path of the book is sketched in Figure 2b.

![Figure 2b](image)

\( f \) is opposite to \( s \), so \( W = -fs = -(3.675 \text{ N})(11.3 \text{ m}) = -42 \text{ J} \)

(c) For the motion from you to Kim

\[
W = -fs = -(3.675 \text{ N})(8.0 \text{ m}) = -29.4 \text{ J}
\]

The total work for the round trip is \(-29.4 \text{ J} - 29.4 \text{ J} = -59 \text{ J} \).

(d) **Evaluate:** Parts (a) and (b) show that for two different paths between you and Carlos, the work done by friction is different. Part (c) shows that when the starting and ending points are the same, the total work is not zero. Both these results show that the friction force is nonconservative.

7.34. **Identify:** Apply \( F(x) = -\frac{dU(x)}{dx} \).

**Set Up:**

\[
\frac{d(1/x)}{dx} = -\frac{1}{x^2}
\]

**Execute:**

\[
F_s(x) = -\frac{d(-Gm_1m_2/x)}{dx} = \frac{Gm_1m_2}{x^2} \frac{d(1/x)}{dx} = \frac{Gm_1m_2}{x^2} \cdot \frac{1}{x^2} = \frac{Gm_1m_2}{x^4}
\]

The force on \( m_2 \) is in the \(-x\)-direction. This is toward \( m_1 \), so the force is attractive.

**Evaluate:** By Newton's 3rd law the force on \( m_1 \) due to \( m_2 \) is \( Gm_1m_2/x^2 \), in the \(+x\)-direction (toward \( m_2 \)). The gravitational potential energy belongs to the system of the two masses.

7.38. **Identify:** Apply Eq. (7.16).

**Set Up:** \( \frac{dU}{dx} \) is the slope of the \( U \) versus \( x \) graph.

**Execute:**

(a) Considering only forces in the \( x \)-direction, \( F_x = -\frac{dU}{dx} \) and so the force is zero when the slope of the \( U \) vs \( x \) graph is zero, at points \( b \) and \( d \).

(b) Point \( b \) is at a potential minimum; to move it away from \( b \) would require an input of energy, so this point is stable.

(c) Moving away from point \( d \) involves a decrease of potential energy, hence an increase in kinetic energy, and the marble tends to move further away, and so \( d \) is an unstable point.

**Evaluate:** At point \( b \), \( F_x \) is negative when the marble is displaced slightly to the right and \( F_x \) is positive when the marble is displaced slightly to the left, the force is a restoring force, and the equilibrium is stable. At point \( d \), a small displacement in either direction produces a force directed away from \( d \) and the equilibrium is unstable.
8.12. IDENTIFY: Apply Eq. 8.9 to relate the change in momentum of the momentum to the components of the average force on it.

SET UP: Let +x be to the right and +y be upward.

EXECUTE: (a) \( J_x = \Delta p_x = m v_{x_f} - m v_{x_i} = (0.145 \text{ kg})(-65.0 \text{ m/s} \cos 30^\circ - 50.0 \text{ m/s}) = -15.4 \text{ kg} \cdot \text{m/s} \).

The horizontal component is 15.4 kg \cdot m/s, to the left, and the vertical component is 4.71 kg \cdot m/s, upward.

(b) \( F_{av-x} = \frac{J_x}{\Delta t} = \frac{-15.4 \text{ kg} \cdot \text{m/s}}{1.75 \times 10^{-3} \text{ s}} = -8800 \text{ N} \).

The horizontal component is 8800 N, to the left, and the vertical component is 2690 N, upward.

EVALUATE: The ball gains momentum to the left and upward and the force components are in these directions.

8.34. IDENTIFY: There is no net external force on the system of the two skaters and the momentum of the system is conserved.

SET UP: Let object \( A \) be the skater with mass 70.0 kg and object \( B \) be the skater with mass 65.0 kg. Let +x be to the right, so \( v_{Ax} = +2.00 \text{ m/s} \) and \( v_{Bx} = -2.50 \text{ m/s} \). After the collision the two objects are combined and move with velocity \( v_x \). Solve for \( v_x \).

EXECUTE: \( P_x = P_{Ax} + P_{Bx} = m_m v_{Ax} + m_m v_{Bx} = (m_A + m_B) v_x \).

\( v_x = \frac{m_A v_{Ax} + m_B v_{Bx}}{m_A + m_B} = \frac{(70.0 \text{ kg})(2.00 \text{ m/s}) + (65.0)(-2.50 \text{ m/s})}{70.0 \text{ kg} + 65.0 \text{ kg}} = -0.167 \text{ m/s} \).

The two skaters move to the left at 0.167 m/s.

EVALUATE: There is a large decrease in kinetic energy.

8.44. IDENTIFY and SET UP: Without rounding, the calculation in Example 8.12 gives \( v_{y2} = \sqrt{20} \text{ m/s} \).

EXECUTE: The two equations in Example 8.12 for \( \alpha \) and \( \beta \) are

\[
(0.500 \text{ kg})(4.00 \text{ m/s}) = (0.500 \text{ kg})(2.00 \text{ m/s})(\cos \alpha) + (0.300 \text{ kg})(\sqrt{20} \text{ m/s})(\cos \beta) \quad \text{Eq. 1}
\]
and

\[
0 = (0.500 \text{ kg})(2.00 \text{ m/s})(\sin \alpha) - (0.300 \text{ kg})(\sqrt{20} \text{ m/s})(\sin \beta) \quad \text{Eq. 2}.
\]

Dividing each equation by \( (0.500 \text{ kg})(1.00 \text{ m/s}) \) gives

\[
4.00 = 2.00 \cos \alpha + 0.6\sqrt{20} \cos \beta \quad \text{Eq. 3}
\]
and

\[
0 = 2.00 \sin \alpha - 0.6\sqrt{20} \sin \beta \quad \text{Eq. 4}.
\]

Eq. 3 gives \( \cos \beta = \frac{4.00 - 2.00 \cos \alpha}{0.6\sqrt{20}} \) and \( \cos^2 \beta + \cos^2 \beta = 2.222 - 2.222 \cos \alpha + 0.5556 \cos^2 \alpha \).

Eq. 4 gives \( \sin \beta = 0.7454 \sin \alpha \) and \( \sin^2 \beta = 0.5556 \sin^2 \alpha = 0.5556 - 0.5556 \cos^2 \alpha \).

Adding the two equations and using \( \sin^2 \beta + \cos^2 \beta = 1 \) gives \( 1 = 2.778 - 2.222 \cos \alpha \) and \( \cos \alpha = 0.8002 \). Then \( \sin \beta = 0.7454 \sin \alpha \) gives \( \beta = 36.9^\circ \).

EVALUATE: For these values of \( \alpha \) and \( \beta \), the x component of momentum, the y component of momentum and the kinetic energy are all conserved in the collision.

8.50. IDENTIFY: Apply Eqs. 8.28, 8.30 and 8.32. There is only one component of position and velocity.

SET UP: \( m_A = 1200 \text{ kg} \), \( m_B = 1800 \text{ kg} \). \( M = m_A + m_B = 3000 \text{ kg} \). Let +x be to the right and let the origin be at the center of mass of the station wagon.
EXECUTE:  
(a) \[ x_{cm} = \frac{m_A x_A + m_B x_B}{m_A + m_B} = \frac{0 + (1800 \text{ kg})(40.0 \text{ m})}{1200 \text{ kg} + 1800 \text{ kg}} = 24.0 \text{ m}. \]

The center of mass is between the two cars, 24.0 m to the right of the station wagon and 16.0 m behind the lead car.

(b) \[ P = m_A v_{A,i} + m_B v_{B,i} = (1200 \text{ kg})(12.0 \text{ m/s}) + (1800 \text{ kg})(20.0 \text{ m/s}) = 5.04 \times 10^4 \text{ kg} \cdot \text{m/s}. \]

(c) \[ v_{cm,v} = \frac{m_A v_{A,v} + m_B v_{B,v}}{m_A + m_B} = \frac{(1200 \text{ kg})(12.0 \text{ m/s}) + (1800 \text{ kg})(20.0 \text{ m/s})}{1200 \text{ kg} + 1800 \text{ kg}} = 16.8 \text{ m/s}. \]

(d) \[ P = M v_{cm,v} = (3000 \text{ kg})(16.8 \text{ m/s}) = 5.04 \times 10^4 \text{ kg} \cdot \text{m/s}, \text{ the same as in part (b)}. \]

EVALUATE:  
The total momentum can be calculated either as the vector sum of the momenta of the individual objects in the system, or as the total mass of the system times the velocity of the center of mass.
Note: Referred equations are from your textbook.

9.12. **IDENTIFY:** In part (b) apply the equation derived in part (a).

**SET UP:** Let the direction the propeller is rotating be positive.

**EXECUTE:** (a) Solving Eq. (9.7) for $t$ gives $t = \frac{\omega_z - \omega_{0z}}{\alpha_z}$. Rewriting Eq. (9.11) as $\theta - \theta_0 = \frac{1}{2}(\omega_z + \omega_{0z})t$ and substituting for $t$ gives

$$\theta - \theta_0 = \left(\frac{\omega_z - \omega_{0z}}{\alpha_z}\right)\left(\omega_z + \frac{1}{2}(\omega_z - \omega_{0z})\right) = \frac{1}{2\alpha_z}(\omega_z^2 - \omega_{0z}^2),$$

which when rearranged gives Eq. (9.12).

(b) $\alpha_z = \frac{1}{2}\left(\omega_z + \omega_{0z}\right)$ to calculate $t = 0.500\ \text{s}$. Then $\omega_z = \omega_{0z} + \alpha_z t$ gives $\alpha_z = 8.00\ \text{rad/s}^2$, which agrees with our results in part (b).

**EVALUATE:** We could also use $\theta - \theta_0 = \frac{1}{2}(\omega_z + \omega_{0z})t$ to calculate $t = 0.500\ \text{s}$. Then $\omega_z = \omega_{0z} + \alpha_z t$ gives $\alpha_z = 8.00\ \text{rad/s}^2$, which agrees with our results in part (b).

9.30. **IDENTIFY:** $a_{tan} = r\alpha$, $v = r\omega$ and $a_{rad} = v^2/r$, $\theta - \theta_0 = \omega_{rev}t$.

**SET UP:** When $\alpha_z$ is constant, $\omega_{rev} = \frac{\omega_{0z} + \alpha_z}{2}$. Let the direction the wheel is rotating be positive.

**EXECUTE:** (a) $\alpha = \frac{a_{tan}}{r} = \frac{-10.0 \ \text{m/s}^2}{0.200 \ \text{m}} = -50.0 \ \text{rad/s}^2$

(b) At $t = 3.00\ \text{s}$, $v = 50.0 \ \text{m/s}$ and $\omega = \frac{v}{r} = \frac{50.0 \ \text{m/s}}{0.200 \ \text{m}} = 250 \ \text{rad/s}$ and at $t = 0$,

$$v = 50.0 \ \text{m/s} + (-10.0 \ \text{m/s}^2)(0 - 3.00 \ \text{s}) = 80.0 \ \text{m/s}, \quad \omega = 400 \ \text{rad/s}.$$

(c) $\omega_{rev}t = (325 \ \text{rad/s})(3.00 \ \text{s}) = 975 \ \text{rad} = 155 \ \text{rev}.$

(d) $v = \sqrt{\alpha_{rad}r} = \sqrt{(9.80 \ \text{m/s}^2)(0.200 \ \text{m})} = 1.40 \ \text{m/s}$. This speed will be reached at time $\frac{50.0 \ \text{m/s} - 1.40 \ \text{m/s}}{10.0 \ \text{m/s}} = 4.86 \ \text{s}$ after $t = 3.00 \ \text{s}$, or at $t = 7.86 \ \text{s}$. (There are many equivalent ways to do this calculation.)

**EVALUATE:** At $t = 0$, $a_{rad} = r\omega^2 = 3.20 \times 10^4 \ \text{m/s}^2$. At $t = 3.00 \ \text{s}$, $a_{rad} = 1.25 \times 10^4 \ \text{m/s}^2$. For $a_{rad} = g$ the wheel must be rotating more slowly than at 3.00 s so it occurs some time after 3.00 s.

9.44. **IDENTIFY:** $K = \frac{1}{2}I\omega^2$. Use Table 9.2 to relate $I$ to the mass $M$ of the disk.

**SET UP:** 45.0 rpm = 4.71 rad/s. For a uniform solid disk, $I = \frac{1}{2}MR^2$.

**EXECUTE:** (a) $I = 2K\omega^2 = \frac{2(0.250 \ \text{J})}{(4.71 \ \text{rad/s})^2} = 0.0225 \ \text{kg} \cdot \text{m}^2$.

(b) $I = \frac{1}{2}MR^2$ and $M = \frac{2I}{R^2} = \frac{2(0.0225 \ \text{kg} \cdot \text{m}^2)}{(0.300 \ \text{m})^2} = 0.500 \ \text{kg}$.

**EVALUATE:** No matter what the shape is, the rotational kinetic energy is proportional to the mass of the object.

9.56. **IDENTIFY:** Using the parallel-axis theorem to find the moment of inertia of a thin rod about an axis through its end and perpendicular to the rod.

**SET UP:** The center of mass of the rod is at its center, and $I_{cm} = \frac{1}{12}ML^2$.

**EXECUTE:** $I_r = I_{cm} + Md^2 = \frac{M}{12}L^2 + M\left(\frac{L}{2}\right)^2 = \frac{M}{3}L^2$.

**EVALUATE:** $I$ is larger when the axis is not at the center of mass.